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THE EFFECTS OF MASS ADDITION ON THE LAMINAR BOUNDARY-LAYER FLOW OF AN ABSORBING–EMITTING GAS

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NOMENCLATURE

E,	Eckert number;
$G(\lambda)$,	radiative correction function defined by equation (9);
$H(\lambda, \epsilon_w)$,	radiative correction function defined by equation (8);
Nu,	Nusselt number;
Pr,	Prandtl number;
q^R,	radiative heat flux per unit area;
Re,	Reynolds number;
T,	temperature;
u,	x-component of velocity;
v,	y-component of velocity;
x,	coordinate along surface of plate;
y,	coordinate normal to plate.

Greek symbols

α,	absorption coefficient;
δ,	boundary-layer thickness;
ϵ,	emissivity;
ζ,	$2\alpha\, xRe^{\frac{1}{2}}$;
η,	Blasius variable;
λ,	wall temperature ratio;
ν,	kinematic viscosity;
ξ,	$2\sigma\alpha T_\infty^3 x/\rho c_p U_\infty$;
ρ,	density;
σ,	Stefan–Boltzmann constant;
τ,	optical thickness.

Subscripts

w,	surface of plate;
δ,	edge of boundary layer;
∞,	free stream.

INTRODUCTION

It is the purpose of the present investigation to extend Cess' work on radiation boundary-layer flow to include the effect of viscous dissipation and mass addition on the temperature field and heat-transfer rates. A steady laminar flow of a diffuse, absorbing and emitting gas having a uniform free stream velocity (U_∞) and temperature (T_∞) is considered to pass over a gray, isothermal flat plate. Gas is injected through the plate at a rate proportional to $X^{-\frac{1}{2}}$ and is assumed to have the same properties as the free stream gas. The analysis further assumes the gas to be non-scattering and absorption coefficient independent of wavelength and temperature.

Varying the Eckert number results in considering two different sets of thermal boundary conditions which correspond to the heated plate and the cooled plate problems. The governing conservations equation corresponding to this problem may be put in the form [1]

$$f'''(\eta) + f''(\eta)\, f'(\eta) = 0 \tag{1}$$

$$\frac{1}{Pr}\Theta'' + f\Theta' + \tfrac{1}{4}E(f'')^2 - 2f\frac{\partial \Theta}{\partial \xi}\xi = -4\zeta[\epsilon_w(\lambda^4 - 1) + (1 + \Theta^4)]. \tag{2}$$

Here E is the Eckert number, Pr the Prandtl number, η a transformed Blasius variable and

$$\Theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \lambda = \frac{T}{T_\infty}, \qquad \xi = \frac{2\sigma\alpha T_\infty^3 X}{\rho c_p u_\infty}.$$

The boundary conditions are

$$\eta = 0 \qquad f'(0) = 0 \qquad f(0) = v_0 \qquad \Theta = 1 \tag{3}$$

$$\eta \to \infty \qquad f'(\eta) \to 2 \qquad \Theta \to \frac{\lambda^4 - 1}{\lambda - 1}\epsilon_w \xi. \tag{4}$$

The solution to equations (1) and (3) with the blowing velocity varying as $X^{-\frac{1}{2}}$ has been carried out by Emmons and Leigh [2] and this tabulated data on $f(\eta)$ will be utilized in the solution of the energy equation. In order to separate out the influences of wall emissivity, assume a solution of Θ to be of the form

$$\Theta = \theta_0 + \frac{\lambda^4 - 1}{\lambda - 1}(\theta_1 + (\epsilon_w - 1)\phi)\,\xi + \ldots \tag{5}$$

Substituting this and the Blasius function for velocity into equation (2) and collecting like powers of ξ, the following simultaneous set of equations arise

$$\frac{1}{Pr}\Theta_0'' + f\theta_0 = -E(f'')^2 \tag{6a}$$

$$\frac{1}{Pr}\Theta_1'' + f\theta_1' - 2f'\theta_1$$

$$= -\frac{4}{\lambda^4 - 1}(\lambda^4 + 1 - 2[1 + (\lambda - 1)\theta_0^4]) \quad (6b)$$

$$\frac{1}{Pr}\phi_1'' + f\phi_1' - 2f\phi_1 = -4 \quad (6c)$$

with the boundary conditions

$$\begin{array}{llll} \eta = 0 & \theta_0 = 1, & \theta_1 = 0, & \phi_1 = 0 \\ \eta \to \infty & \theta_0 = 0, & \theta_1 = 1, & \phi_1 = 1. \end{array} \quad (6d)$$

Equation (6a) describes the temperature field in the absence of radiation effects and has the well-known solution by Pohlhausen. The solution to equation (6a) is tabulated in the literature but as a convenience for the machine computation it was integrated numerically. The first-order radiation effects governed by (6b) and (6c) were also integrated numerically using a central difference technique to give the temperature distribution in the thermal boundary layer.

The temperature distribution in the entire fluid is obtained by combining the temperature distribution in the radiation layer with equation (5), which yields

$$\Theta = \theta_0 + \left(\frac{\lambda^4 - 1}{\lambda - 1}\right)[\theta_1 + (\epsilon_w - 1)\phi_1]E_2(\tau)\,\xi. \quad (7)$$

Once the temperature distribution has been found, the heat transfer to the surface may be evaluated. Following [1] the convective Nusselt number may be expressed as

$$\frac{Nu}{\sqrt{(Re)}} = -\tfrac{1}{2}(\theta_0'(0)$$

$$+ (\lambda^4 - 1/\lambda - 1)[\theta_1'(0) + (\epsilon_w - 1)\phi_1(0)]\,\xi$$

$$= -\tfrac{1}{2}\theta_0'(0) + H(\lambda, \epsilon_w)\,\xi \quad (8)$$

and the first-order radiative flux to the plate is given by

$$\frac{q^R}{\epsilon_w\sigma(T_w^4 - T_\infty^4)} = 1 - \frac{1}{\lambda^4 - 1}\int_0^\infty ([1 + (\lambda - 1)\theta_0]^4 - 1)\,d\eta\,\zeta$$

$$= 1 - G(\lambda)\,\zeta \quad (9)$$

where

$$\zeta \equiv \frac{2\alpha x}{\sqrt{(Re)}} \approx \tau.$$

DICUSSION AND CONCLUSIONS

The typical temperature distribution for a hot wall case is given in Fig. 1 for various values of E. Shown for comparison is the distribution for no radiative coupling ($\xi = 0$) and very slight coupling ($\xi = 0.005$). For the case shown ($E = 9$, $\lambda = 2$, $\zeta = 0.01$, $f(0) = -0.4$), the effect of radiation is to

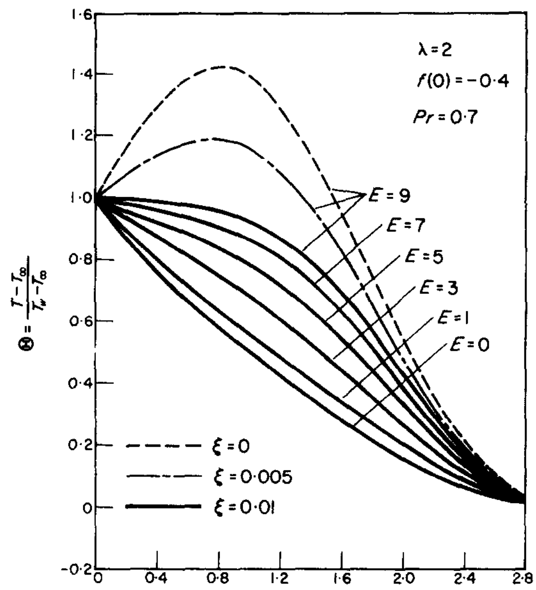

FIG. 1. Temperature distribution.

reduce the temperature gradient such that an adiabatic profile is essentially obtained. For the cold wall case, the results indicate that mass addition has a strong influence on the radiative correction term $H(\lambda, \epsilon_w)$ (Fig. 2). At large values of $E(E \approx -9)$, this correction term, $H(\lambda, \epsilon_w)$, can change signs over the range of blowing rates considered and hence the radiation may either increase or decrease the convective heat transfer, depending on the blowing rate and value of E.

In contrast with this, the results for the hot wall case shows that blowing increases $H(\lambda, \epsilon_w)$ for small values of the Eckert number, while for large values of E, $H(\lambda, \epsilon_w)$ is insensitive to small injection rates but is reduced for moderate to large mass injection (Fig. 3).

For the larger values of E considered in the cold wall case, (Fig. 4), the first-order radiative correction term $G(\lambda)$ is seen to be insensitive to small rates of mass injection but greatly altered for large mass injection. For smaller values of the Eckert number, $G(\lambda)$ is increased to some degree over the entire range of blowing rates considered (thus reducing the radiant heat transfer). In the hot wall case, the radiation correction term $G(\lambda)$ is always positive but it exhibits a maximum (Fig. 5). This maximum tends to shift towards a lower blowing velocity as the Eckert number is increased. Thus, increased blowing initially reduces the net radiant heating (G increases) up to an "optimal" blowing velocity after which the radiation correction term G rapidly decreases with the result that the radiant heating may be

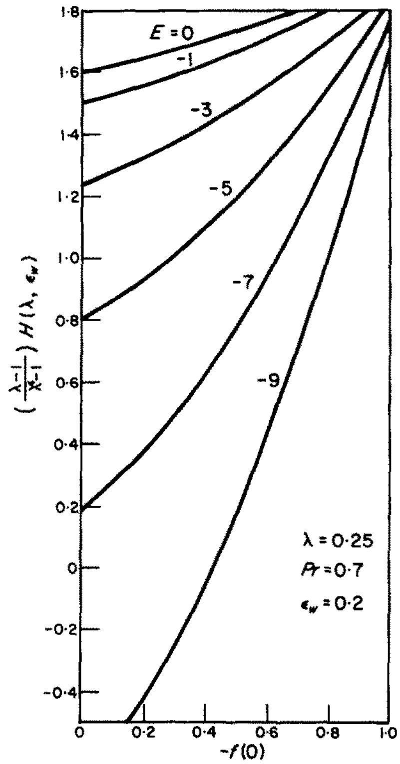

FIG. 2. Correction term $H(\lambda, \epsilon_w)$.

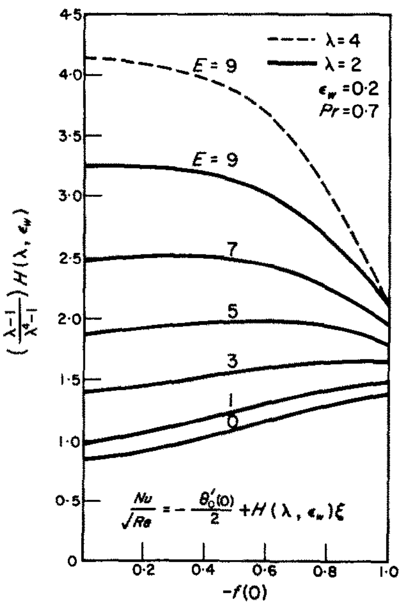

FIG. 3. Correction term $H(\lambda, \epsilon_w)$.

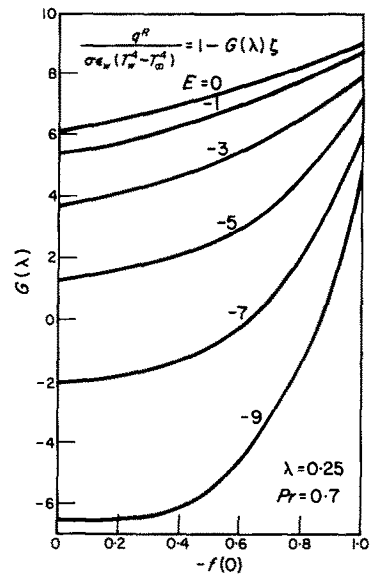

FIG. 4. Correction term $G(\lambda)$.

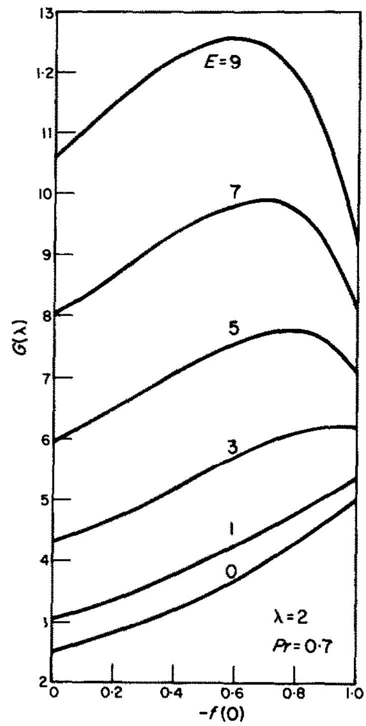

FIG. 5. Correction term $G(\lambda)$.

increased over that with no mass injection. Hence the net radiant heating exhibits a minimum value.

In summary, it is concluded from this study that for the cold wall case, the radiative component of heat transfer is relatively insensitive to small blowing velocities but can be altered significantly at moderate to large blowing rates. The first-order radiation correction term of the convective heating was found to be greatly increased over the whole range of blowing considered.

For the hot wall case, the radiative correction term $G(\lambda)$ exhibits a maximum for a given Eckert number and blowing velocity. The total radiative component is altered over the entire range of blowing rates considered and a minimum value is found to exist [for a given E and $f(0)$]. The first-order radiative correction term of the convective heating was found to be insensitive to small values of $f(0)$ while at moderate to large blowing rates, this correction term increased for small values of the Eckert number and decreased for large values of E as the blowing increased. Additional results supporting this conclusion may be found in [3].

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SHIELDING OF RADIATION BY SCREENS AND ITS SIMILARITY TO A GRAY GAS

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NOMENCLATURE

a, a length given in Fig. 2;
b, a length given in Fig. 2;
B, black body intensity;
d, diameter given in Fig. 2;
e, black body emissive power;
F_m, a function defined by equation (4);
G_m, a function defined by equation (4);
H_m, a function defined by equation (6);
I, intensity of radiation;
n, number of screens;
q, heat flux;
R, radiosity.

Greek symbols

α, absorptivity;
δ, distance between two neighboring screens for equally spaced screens;
θ, polar angle measured from normal, between (I^+, τ) or $(I^-, -\tau)$;
κ, volumetric absorption coefficient;
ξ, τ/τ_∞;
τ, optical thickness measured from lower wall, $= \alpha_n i$;
ϕ, dimensionless emissive power $(e_i - R_w)/(R_\infty - R_w)$;
ω, solid angle.

Subscripts and superscripts

∞, upper wall;
i, screen number;
k, dummy index;
m, dummy index;
n, number of screens, also normal incidence;
w, lower wall;
θ, in the θ direction;
$+$, positive τ direction;
$-$, negative τ direction.